LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

SECOND SEMESTER – APRIL 2023

16/17/18UMT2AL01 – MATHEMATICS FOR PHYSICS - II

Date: 15-05-2023   Dept. No.   Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

PART-A

Answer ALL questions   10x2=20 marks

1. Evaluate $\int (4x^3 + 3x^2 - 2x + 5)dx$.

2. Find the value $\int_0^{\frac{\pi}{2}} cos^6 x dx$.

3. State any two properties of gamma function.

4. Prove that $\beta(n, m) = \beta(m, n)$.

5. If α and β are the real and distinct roots of differential equation, write the complementary function.

6. Write the criterion for to be exact $Mdx + Ndy = 0$.

7. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyzdxdydz$.

8. Evaluate $\int_0^a \int_0^b xy(x - y)dxdy$.

9. Find the directional derivative of $\emptyset = x^2yz + y^2z$ at (1,-2,-1) in the direction of
   $2\vec{\imath} + \vec{\jmath} - 3\vec{k}$.

10. Prove that $\nabla.\vec{r} = 0$.

PART – B

Answer any FIVE questions   5x8=40 marks

11. Evaluate $\int \frac{3x+1}{(x-1)^3(x+3)} dx$.

12. Establish the reduction formula $I_n = \int cos^n x dx$.

13. Solve $I = \int_0^{\frac{\pi}{2}} logsinxdx$.

14. Show that $\frac{2^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}} = 1.3.5 \dots (2n - 1)$.

15. Solve $(D^2 - 4D + 3)y = e^{-x}sinx$.

16. Solve $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$.

17. Find divergence and curl of the vector $F = xyz \vec{\imath} + 3x^2y\vec{\jmath} + (xz^2 - y^2z)\vec{k})$ at $(1,2,-1)$.

18. Evaluate $\iint_s F.\hat{n}ds$ where F = zi +xj –y^2z k, and S is the surface of the cylinder $x^2 +y^2 =1$ included in the first octant between the planes z=0 and z=2.

PART C

2x20=40 marks

19. (a) Prove that $\int_0^{\frac{\pi}{2}} \log(1 + tan\theta) \, d\theta = \frac{\pi}{8} log2$.   (10)

    (b) Solve $(3D^2 + D - 14)y = 13e^{2x}$.   (10)

20. (a) Derive the reduction formula $I_n = \int sin^n x dx$.   (10)

    (b) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.   (10)

21. (a)By changing into polar co-ordinates evaluate the integral $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)dxdy$.

    (b) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}dxdy$.   (10+10)

22. (a) Evaluate $\int \frac{3x+7}{2x^2+3x-2} dx$.   (10)

    (b) Find by Green's theorem the value of $\int_c (x^2 ydx + ydy)$ along the closed curve C formed by the
        curves y^2=x and y=x between (0, 0) and (1, 1).   (10)

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